Nusselt Number, Stanton Number and Skin-Friction Coefficient: Physical Meaning and Engineering Use

1 Introduction

Thermal-fluid design is governed by two wall quantities: the wall heat flux q''_{w} and the wall shear stress τ_{w}. The first sets heat-exchanger size, turbine-blade metal temperature and the temperature margin of electronic components; the second sets drag, pressure drop and pumping power. The heat flux is conventionally written as

$$q''_{\mathrm{w}} = h\,\Delta\Theta, \qquad \Delta\Theta = \Theta_{\mathrm{w}} - \Theta_{\infty}, \tag{1}$$

which transfers the difficulty to the heat-transfer coefficient h. Unlike k or μ, h is not a property of the fluid. It depends on the flow and the geometry,

$$h = F(U,\, L,\, \rho,\, \mu,\, k,\, c_p). \tag{2}$$

Characteristic scales

The quantities U and L in (2) are the characteristic velocity and the characteristic length. Both are fixed by the problem specification before any solution is known. Table 1 lists the conventional choices; the subscript of a dimensionless group identifies the length used.

Table 1: Conventional characteristic scales.

Configuration	L	U	Groups
Flat plate, local	distance from the leading edge, x	free-stream velocity U_∞	Re_x, Nu_x
Flat plate, averaged	plate length L	U_∞	Re_L, $\overline{Nu}_L$
Pipe or duct	diameter D, or hydraulic diameter $D_h = 4A/P$	bulk velocity U_b	Re_D, Nu_D
Cylinder, sphere	diameter D	U_∞	Re_D, Nu_D
Turbine blade	chord c	inlet or exit velocity	Re_c, Nu_c

A length is required because, without L, the variables U, ρ, μ, k, c_p form a single dimensionless group, the Prandtl number, and neither h nor τ_{w} can be made dimensionless. The geometry is the only source of a length known in advance. Boundary-layer thicknesses have the dimension of length, but they are results of the flow and cannot serve as inputs.

By convention L is the length that controls the development of the flow. For a semi-infinite plate the distance x is the only geometric length and the boundary layer grows with it; in fully developed duct flow the profiles are independent of x and are set by D. Replacing one geometric length by another (a radius for a diameter, for example) changes the groups by constant factors, so a correlation is valid only with the length used in its derivation.

Dimensional analysis

Equation (2) contains seven variables in four dimensions (M, L, T, Θ). By the Buckingham Π theorem it reduces to a relation among three dimensionless groups, for example

$$Nu = f(Re, \mathrm{Pr}). \tag{3}$$

Similarly, $\tau_\mathrm{w} = G(U, L, \rho, \mu)$ reduces to $C_f = g(Re)$. A correlation measured on a model therefore applies to any geometrically similar configuration at the same Re and Pr.

2 Heat transfer mechanisms

Definitions

Conduction Transport of energy by random molecular motion: molecular collisions in gases, lattice vibrations and intermolecular forces in liquids and solids, and free electrons in metals. No bulk motion is required. The flux is given by Fourier's law, $\mathbf{q} = -k\nabla\Theta$.

Advection Transport of enthalpy by the bulk motion of the fluid. The flux relative to a reference temperature is $\mathbf{q} = \rho c_p \mathbf{u}\,(\Theta - \Theta_\mathrm{ref})$.

Convection The combined action of advection and conduction in a moving fluid. Convective heat transfer denotes the exchange of heat between a surface and a moving fluid, characterised by h.

For steady flow with constant properties, the energy equation expresses the balance between the two mechanisms,

$$\underbrace{\rho c_p \left(u\frac{\partial\Theta}{\partial x} + w\frac{\partial\Theta}{\partial z} \right)}_{\text{advection}} = \underbrace{k\left(\frac{\partial^2\Theta}{\partial x^2} + \frac{\partial^2\Theta}{\partial z^2} \right)}_{\text{conduction}} + \underbrace{\mu\Phi}_{\text{viscous dissipation}}, \tag{4}$$

where x is the streamwise coordinate, z the wall-normal coordinate, and u, w the corresponding velocity components. Viscous dissipation converts kinetic energy into internal energy. It is a volumetric heat source, not a transport mechanism, and is neglected in what follows. Turbulent transport is advection by velocity fluctuations; it is treated in Section 5.

Heat transfer at the wall

At the wall $u = w = 0$, and velocity fluctuations also vanish. The advective flux is therefore zero, in laminar and in turbulent flow, and the wall heat flux is exactly

$$q''_\mathrm{w} = -k \left.\frac{\partial\Theta}{\partial z}\right|_{z=0}. \tag{5}$$

Heat crosses the wall by conduction only. Fluid motion affects q''_w solely through the wall temperature gradient: advection removes heated fluid from the near-wall region and replaces it with fluid at the free-stream temperature, which maintains a steep gradient. The prediction of convective heat transfer is therefore the prediction of the wall temperature gradient.

Molecular origin of the Prandtl number

In a dilute gas the same molecular motion transports momentum and energy. Elementary kinetic theory gives $\nu \sim \alpha \sim \frac{1}{3}\bar{c}\lambda$, where $\bar{c}$ is the mean molecular speed and λ the mean free path, so that $\mathrm{Pr} \approx 1$. The Eucken correction gives $\mathrm{Pr} = 4\gamma/(9\gamma - 5)$, equal to 0.74 for air ($\gamma = 1.4$) and 0.67 for monatomic gases. In other fluids the carriers differ (Table 2).

Table 2: Prandtl number and dominant carriers.

Fluid	Pr	Carriers of momentum and heat
Gases	≈ 0.7	The same molecules carry both.
Water (20 °C)	≈ 7	Hydrogen bonding raises the viscosity.
Oils	10^2–10^4	Long molecules transmit momentum strongly; conduction is modest.
Liquid metals	0.004–0.03	Free electrons carry heat but almost no momentum.

3 Boundary-layer thicknesses

Two kinds of thickness are used. The profile thicknesses locate the region in which the profiles change:

- the velocity boundary layer δ, at which $u = 0.99\,U_\infty$;
- the thermal boundary layer δ_θ, at which $(\Theta - \Theta_\mathrm{w})/(\Theta_\infty - \Theta_\mathrm{w}) = 0.99$.

The wall-gradient thicknesses are defined by the tangents to the profiles at the wall:

$$\delta_q \equiv \frac{\Delta\Theta}{-(\partial\Theta/\partial z)_\mathrm{w}} = \frac{k}{h}, \qquad \delta_\tau \equiv \frac{U_\infty}{(\partial u/\partial z)_\mathrm{w}} = \frac{\mu U_\infty}{\tau_\mathrm{w}}. \tag{6}$$

The conduction-layer thickness δ_q is the thickness of a stagnant fluid layer that, with $\Delta\Theta$ across it, would conduct the actual wall heat flux. The wall-shear thickness δ_τ is the thickness of a layer of uniform shear τ_w that would produce the velocity difference U_∞. The term wall-shear thickness is used to avoid confusion with free shear layers such as mixing layers and jets.

Only δ_q and δ_τ enter h and τ_w. Table 3 compares the four thicknesses for a laminar flat plate in air, and for the turbulent case of Section 8.

Table 3: Boundary-layer thicknesses.

	δ	δ_τ	δ_θ	δ_q
Laminar, Pr = 0.7 (units of $x\,Re_x^{-1/2}$)	4.91	3.01	5.64	3.42
Turbulent, Section 8 (mm)	22	0.43	–	0.48

4 Reference fluxes and dimensionless groups

A wall flux is made dimensionless by dividing it by a reference flux formed from the characteristic scales. Each reference flux corresponds to a single transport mechanism acting alone.

Conduction reference

In a stagnant fluid slab with temperature difference $\Delta\Theta$ across it, equation (4) reduces to $d^2\Theta/dz^2 = 0$. The profile is linear and Fourier's law gives the conducted flux as $k\,\Delta\Theta$ divided by the slab thickness. A slab of thickness equal to the characteristic length defines the conduction reference,

$$q_\mathrm{cond} \equiv \frac{k\,\Delta\Theta}{L}. \tag{7}$$

Writing the actual wall flux in the same form recovers the conduction-layer thickness of (6), $q''_{\mathrm{w}} = k\,\Delta\Theta/\delta_q$. Both expressions describe conduction through a stagnant slab; L is known before the problem is solved, whereas δ_q is fixed by the flow. Their ratio is the Nusselt number,

$$Nu = \frac{hL}{k} = \frac{q''_{\mathrm{w}}}{q_{\mathrm{cond}}} = \frac{L}{\delta_q}. \tag{8}$$

Nu is the number of conduction-layer thicknesses contained in the characteristic length. On a flat plate $Nu_x = x/\delta_q$ increases downstream because x grows faster than δ_q. In fully developed laminar pipe flow $\delta_q \propto D$ and Nu_D is constant.

Advection reference

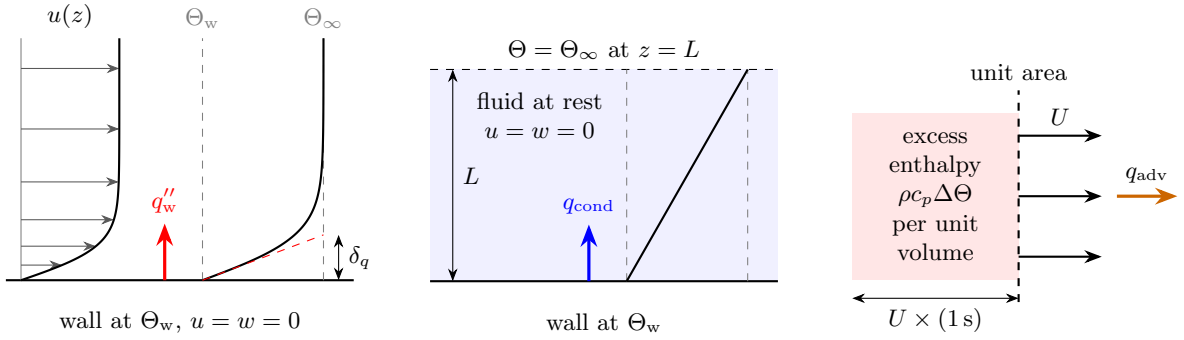
Fluid moving at velocity U carries an excess enthalpy $\rho c_p \Delta\Theta$ per unit volume through a volume flux U per unit area. The advection reference is

$$q_{\mathrm{adv}} \equiv \rho c_p\, U\, \Delta\Theta. \tag{9}$$

It contains no length and represents the rate at which the stream supplies its temperature excess. The Stanton number is

$$St = \frac{h}{\rho c_p U} = \frac{q''_{\mathrm{w}}}{q_{\mathrm{adv}}}. \tag{10}$$

The numerator shared by Nu and St and the two reference fluxes are illustrated in Figure 1.



(a) Actual flow: numerator of Nu and St

(b) Reference for Nu: conduction without flow

(c) Reference for St: advective supply of the stream

Figure 1: Numerator and denominators of the Nusselt and Stanton numbers. (a) Actual flow: the fluid is at rest at the wall and in motion above it. The wall heat flux crosses the wall by conduction, and the temperature drop is confined to a layer of effective thickness δ_q (tangent construction, dashed). (b) Conduction reference: the same fluid without flow, with the same $\Delta\Theta$ applied across a layer of thickness L, giving $q_{\mathrm{cond}} = k\Delta\Theta/L$. (c) Advection reference: in unit time a column of the stream of length U, carrying an excess enthalpy $\rho c_p \Delta\Theta$ per unit volume, crosses unit area normal to the flow, giving $q_{\mathrm{adv}} = \rho c_p U \Delta\Theta$. $Nu = (a)/(b) = L/\delta_q$ and $St = (a)/(c)$. Not to scale: in the example of Section 8, $L/\delta_q \approx 2000$.

Momentum references

A slab of thickness L with a linear velocity profile (plane Couette flow) carries the viscous reference stress $\tau_{\mathrm{visc}} \equiv \mu U/L$; writing the wall stress in the same form, $\tau_{\mathrm{w}} = \mu U/\delta_\tau$, recovers the wall-shear thickness. Momentum per unit volume ρU transported by a volume flux U gives the advective reference $\tau_{\mathrm{adv}} \equiv \rho U^2$.

Summary of the groups

Table 4: Dimensionless groups as ratios of fluxes.

Group	Ratio	Interpretation
$Nu = hL/k$	$q''_{\mathrm{w}}/q_{\mathrm{cond}} = L/\delta_q$	Wall heat flux relative to conduction across L
$St = h/(\rho c_p U)$	$q''_{\mathrm{w}}/q_{\mathrm{adv}}$	Wall heat flux relative to the advective enthalpy flux of the stream
$Pe = UL/\alpha$	$q_{\mathrm{adv}}/q_{\mathrm{cond}}$	Advection relative to conduction in the fluid
$C_f/2 = \tau_{\mathrm{w}}/(\rho U^2)$	$\tau_{\mathrm{w}}/\tau_{\mathrm{adv}}$	Momentum counterpart of St
$(C_f/2)\,Re$	$\tau_{\mathrm{w}}/\tau_{\mathrm{visc}} = L/\delta_\tau$	Momentum counterpart of Nu
$Re = UL/\nu$	$\tau_{\mathrm{adv}}/\tau_{\mathrm{visc}}$	Momentum counterpart of Pe
$\mathrm{Pr} = \nu/\alpha$	Pe/Re	Fluid property

Every group built on a conduction or viscous reference (Nu, Pe, Re, $(C_f/2)Re$) contains L. St and $C_f/2$ are built on advective references only and contain no length.

Interpretation of Nu and St

The numerators of Nu and St are the same quantity, the wall heat flux, which by (5) is a conductive flux. The two groups differ only in the reference against which this flux is measured. Neither group is a ratio involving convection.

Nu is not the ratio of convection to conduction. This description is common but incorrect. Nu is the ratio of two conductive fluxes (Figure 1a,b): conduction at the wall in the actual flow, where the fluid is at rest at the wall and in motion above it; and conduction through the same fluid without flow, with the same $\Delta\Theta$ applied across a layer of thickness L. In both cases the velocity at the wall is zero. Nu measures the increase in the wall heat flux produced by the motion of the fluid above the wall, which steepens the temperature gradient at the wall; $Nu = 1$ means no increase, as in a fluid layer heated from below before the onset of convection (Rayleigh–Bénard), where $Nu = 1$ exactly. The ratio of advection to conduction in the fluid is the Péclet number.

St is not the ratio of convection to thermal capacity. St is often described as the ratio of heat transferred to the thermal capacity of the fluid. The volumetric heat capacity ρc_p [$\mathrm{J\,m^{-3}\,K^{-1}}$] is a stored quantity and cannot be compared with a flux. The denominator of St is the advective enthalpy flux $\rho c_p U \Delta\Theta$, that is, the heat capacity transported at velocity U (Figure 1c); the numerator is the conductive wall flux. St is therefore the fraction of the enthalpy supplied by the stream that is conducted into the wall.

Nusselt and Biot numbers. Nu uses the conductivity of the fluid. The Biot number hL/k_{s} has the same form but uses the conductivity of the solid; it compares the fluid-side conductance with conduction within the solid and addresses a different question.

Role of each group. Pe is an input, known before the problem is solved. Nu and St are outcomes and are two expressions of the same h. St is the more direct measure when an imposed stream drives the transport.

5 Molecular and turbulent transport

Molecular transport

Each molecular flux is a diffusivity multiplied by the gradient of a quantity per unit volume:

$$\tau = \nu \frac{\partial(\rho u)}{\partial z}, \qquad q = -\alpha \frac{\partial(\rho c_p \Theta)}{\partial z} \quad \Longrightarrow \quad \mu = \rho\nu, \qquad k = \rho c_p \alpha. \tag{11}$$

A transport coefficient is thus a diffusivity multiplied by the capacity of the fluid to hold the transported quantity, and $\mathrm{Pr} = \nu/\alpha = \mu c_p/k$ compares the two diffusivities directly.

Turbulent transport

With the decomposition $u = \bar{u} + u'$, $w = \bar{w} + w'$, $\Theta = \bar{\Theta} + \Theta'$, averaging introduces the Reynolds shear stress $-\rho\overline{u'w'}$ and the turbulent heat flux $\rho c_p \overline{w'\Theta'}$. Both represent advection by velocity fluctuations. The eddy-diffusivity model writes them in the molecular form, $-\overline{u'w'} = \nu_t \, \partial\bar{u}/\partial z$ and $-\overline{w'\Theta'} = \alpha_t \, \partial\bar{\Theta}/\partial z$, so that the total fluxes become

$$\tau = \rho(\nu + \nu_t)\frac{\partial \bar{u}}{\partial z}, \qquad q = -\rho c_p(\alpha + \alpha_t)\frac{\partial \bar{\Theta}}{\partial z}, \qquad \mathrm{Pr}_t = \frac{\nu_t}{\alpha_t}. \tag{12}$$

The effective conductivity is

$$k_{\mathrm{eff}} = \rho c_p(\alpha + \alpha_t) = k\left(1 + \frac{\mathrm{Pr}}{\mathrm{Pr}_t}\frac{\nu_t}{\nu}\right). \tag{13}$$

The eddy diffusivities are the macroscopic counterparts of the molecular ones (Table 5). Because the same eddies transport momentum and heat, $\mathrm{Pr}_t \approx 0.85$–$0.9$ in the logarithmic region of wall flows in air and water. It differs from unity because pressure fluctuations redistribute momentum but not heat. Unlike ν and α, the eddy diffusivities are properties of the flow. Both vanish at the wall, grow approximately as z^3 close to it, and grow linearly in the logarithmic region, $\nu_t \approx \kappa u_\tau z$ with $\kappa \approx 0.41$ and $u_\tau = \sqrt{\tau_{\mathrm{w}}/\rho}$.

Table 5: Molecular and turbulent transport.

	Molecular	Turbulent
Carrier	molecules	eddies
Velocity scale	mean molecular speed $\bar{c}$	fluctuation velocity u'
Length scale	mean free path λ	mixing length ℓ_m
Diffusivity	$\nu \sim \bar{c}\lambda$	$\nu_t \sim u'\ell_m$
Ratio	Pr (fluid property)	Pr_t (flow property)

Series thermal resistance

Near the wall the convective terms are small and the fluxes are approximately constant, $q \approx q''_{\mathrm{w}}$ and $\tau \approx \tau_{\mathrm{w}}$. Integrating the flux laws across the layer under this approximation gives

$$\frac{1}{h} = \int_0^{\delta_\theta} \frac{dz}{\rho c_p(\alpha + \alpha_t)}, \qquad \delta_q = \frac{k}{h} = \int_0^{\delta_\theta} \frac{\alpha}{\alpha + \alpha_t}\, dz, \tag{14}$$

$$St = \left[U_\infty \int_0^{\delta_\theta} \frac{dz}{\alpha + \alpha_t}\right]^{-1}, \qquad \frac{C_f}{2} = \left[U_\infty \int_0^{\delta} \frac{dz}{\nu + \nu_t}\right]^{-1}. \tag{15}$$

The layer acts as a set of thermal resistances in series. Regions where $\alpha_t \gg \alpha$ contribute little to δ_q, so the resistance is concentrated in the viscous sublayer, and the final transfer to the wall is by conduction, consistent with (5).

6 Relations between the groups

Nu = St Re Pr

From the definitions,

$$St = \frac{h}{\rho c_p U} = \frac{hL}{k} \cdot \frac{k}{\rho c_p U L} = Nu\,\frac{\alpha}{UL} = \frac{Nu}{Pe} = \frac{Nu}{Re\,\mathrm{Pr}}. \tag{16}$$

In terms of reference fluxes the identity reads $q''_\mathrm{w}/q_\mathrm{adv} = (q''_\mathrm{w}/q_\mathrm{cond})(q_\mathrm{cond}/q_\mathrm{adv})$. The heat and momentum groups correspond as $St \leftrightarrow C_f/2$, $Nu \leftrightarrow (C_f/2)Re$ and $Pe \leftrightarrow Re$.

Relations in terms of thicknesses

$$Nu_x = \frac{x}{\delta_q}, \qquad \frac{C_f}{2} Re_x = \frac{x}{\delta_\tau}, \qquad \frac{St}{C_f/2} = \frac{1}{\mathrm{Pr}}\,\frac{\delta_\tau}{\delta_q}. \tag{17}$$

Integral balances

With $\Theta' = \Theta - \Theta_\infty$ and continuity, the boundary-layer energy equation can be written in conservative form,

$$\frac{\partial (u\Theta')}{\partial x} + \frac{\partial (w\Theta')}{\partial z} = \alpha \frac{\partial^2 \Theta}{\partial z^2}. \tag{18}$$

Integrating from $z = 0$ to $z \to \infty$, with $w = 0$ at the wall and $\Theta' \to 0$, $\partial\Theta/\partial z \to 0$ in the free stream,

$$\frac{d}{dx}\int_0^\infty u\,\Theta'\,dz = -\alpha\,\frac{\partial \Theta}{\partial z}\bigg|_\mathrm{w} = \frac{q''_\mathrm{w}}{\rho c_p}. \tag{19}$$

For an isothermal wall, division by $U_\infty \Delta\Theta$ gives

$$\frac{d\Delta_2}{dx} = St_x, \qquad \Delta_2 \equiv \int_0^\infty \frac{u}{U_\infty}\,\frac{\Theta - \Theta_\infty}{\Theta_\mathrm{w} - \Theta_\infty}\,dz, \qquad \int_0^x q''_\mathrm{w}\,dx' = \rho c_p U_\infty\,\Delta\Theta\,\Delta_2(x). \tag{20}$$

The enthalpy thickness Δ_2 is the thickness of a free-stream layer at the full temperature excess $\Delta\Theta$ that would carry the same excess enthalpy flux as the boundary layer. All heat added by the wall upstream of x is contained in this flux, and St is the rate at which the layer accumulates it.

The momentum equation, written as

$$\frac{\partial}{\partial x}\big[u(U_\infty - u)\big] + \frac{\partial}{\partial z}\big[w(U_\infty - u)\big] = -\nu \frac{\partial^2 u}{\partial z^2}, \tag{21}$$

integrates in the same way to

$$\frac{d\delta_2}{dx} = \frac{C_{f,x}}{2}, \qquad \delta_2 \equiv \int_0^\infty \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dz, \qquad \int_0^x \tau_\mathrm{w}\,dx' = \rho U_\infty^2\,\delta_2(x). \tag{22}$$

The momentum thickness δ_2 is the thickness of a free-stream layer whose momentum flux equals the momentum deficit of the boundary layer; the drag up to x equals this deficit, which is the basis of drag measurement by wake survey. Both results hold in turbulent flow, since the Reynolds fluxes vanish at the wall and in the free stream. For the Blasius solution, $\delta_2 = 0.664\,x\,Re_x^{-1/2}$, so that $d\delta_2/dx = 0.332\,Re_x^{-1/2} = C_{f,x}/2$.

Duct flow

For a pipe of diameter D with bulk velocity U_b, bulk temperature Θ_b and mass flow rate $\dot{m} = \rho U_b \pi D^2/4$, an energy balance on a slice of length dx gives

$$\dot{m} c_p \, d\Theta_b = q''_{\mathrm{w}} \, \pi D \, dx \quad \Longrightarrow \quad \frac{d\Theta_b}{dx} = \frac{4\,St}{D} \, (\Theta_{\mathrm{w}} - \Theta_b), \tag{23}$$

with $St = h/(\rho c_p U_b)$. For constant Θ_{w} and St,

$$\frac{\Theta_{\mathrm{w}} - \Theta_b(L)}{\Theta_{\mathrm{w}} - \Theta_b(0)} = \exp\!\left(-4\,St\,\frac{L}{D} \right), \tag{24}$$

where $4\,St\,L/D$ is the number of transfer units of the tube. A momentum balance on the same slice gives

$$-\frac{dp}{dx}\,\frac{\pi D^2}{4} = \tau_{\mathrm{w}}\,\pi D \quad \Longrightarrow \quad -\frac{dp}{dx} = \frac{4}{D}\,\frac{C_f}{2}\,\rho U_b^2, \tag{25}$$

with the Darcy friction factor $f = 4C_f$. The temperature change is governed by $St\,L/D$ and the pressure drop by $(C_f/2)\,L/D$.

Mass transfer

The Chilton–Colburn analogy extends the correspondence to mass transfer,

$$j_H = St\,\mathrm{Pr}^{2/3} = \frac{C_f}{2} = j_D = St_m\,Sc^{2/3}, \tag{26}$$

which permits heat-transfer coefficients to be obtained from mass-transfer experiments such as naphthalene sublimation.

7 Choice between St, C_f and Nu

Advantages of St and C_f

1. Both divide a wall flux by the advective flux of the stream. Their ratio is therefore the heat transfer obtained per unit friction, whereas a comparison of Nu with C_f mixes a conduction reference with an advection reference.

2. Both appear directly in the governing balances: St in the growth of the enthalpy thickness and in the outlet temperature of a duct, $C_f/2$ in the growth of the momentum thickness and in the pressure drop.

3. Neither contains a length. On a laminar plate $Nu_x = hx/k$ increases downstream although h decreases, while St_x follows h. In turbulent flow St and C_f vary approximately as $Re^{-0.2}$, compared with $Re^{0.8}$ for Nu.

4. Friction is easier to measure than heat transfer, which was the motivation of Reynolds (1874). Preliminary heating estimates for hypersonic vehicles use $2St/C_f \approx 1.0$–1.2. The Gnielinski–Petukhov correlation for turbulent pipe flow is built on the same ratio,

$$St = \frac{C_f/2}{1.07 + 12.7\sqrt{C_f/2}\,(\mathrm{Pr}^{2/3} - 1)}. \tag{27}$$

   Enhanced surfaces (ribs, dimples, fins) are compared at equal pumping power by the index $(Nu/Nu_0)/(f/f_0)^{1/3}$.

Cases favouring Nu

Nu is the appropriate group when no imposed velocity sets the scale: natural convection, where $Nu = f(Ra, \mathrm{Pr})$; fully developed laminar duct flow, where Nu_D is constant (3.66 for uniform wall temperature and 4.36 for uniform heat flux in a circular tube); liquid metals, where correlations depend on Pe, for example $Nu = 7 + 0.025\,Pe^{0.8}$ (Lyon–Martinelli, uniform heat flux); and thermal-resistance networks, which require $h = Nu\,k/L$ directly.

8 Worked example

Air at 20 m/s and approximately 300 K flows over a plate 1 m long. The boundary layer is assumed turbulent from the leading edge, and quantities are evaluated at $x = L$ (Table 6).

Table 6: Heat transfer predicted from friction.

Quantity	Expression	Value
Reynolds number	$U_\infty x/\nu$	1.32×10^6
Skin friction	$C_f/2 = 0.0296\,Re_x^{-1/5}$	1.77×10^{-3}
Wall shear stress	$(C_f/2)\,\rho U_\infty^2$	0.85 Pa
Stanton number	$(C_f/2)\,\mathrm{Pr}^{-2/3}$, $\mathrm{Pr} = 0.713$	2.21×10^{-3}
Heat-transfer coefficient	$St\,\rho c_p U_\infty$	$54\,\mathrm{W\,m^{-2}\,K^{-1}}$
Nusselt number	$St\,Re_x\mathrm{Pr}$	2080
Conduction-layer thickness	k/h	0.48 mm
Wall-shear thickness	$\mu U_\infty/\tau_\mathrm{w}$	0.43 mm
Boundary-layer thickness	$0.37\,x\,Re_x^{-1/5}$	22 mm

A wall shear stress of 0.85 Pa yields $h \approx 54\,\mathrm{W\,m^{-2}\,K^{-1}}$ without any thermal measurement. The thermal resistance is equivalent to a stagnant film about 2% of the boundary-layer thickness, or approximately 27 viscous units ($\nu/u_\tau \approx 18\,\mu$m). The thicknesses satisfy $(1/\mathrm{Pr})\,\delta_\tau/\delta_q = 1.26$, in agreement with $\mathrm{Pr}^{-2/3} = 1.27$. The relation $St = (C_f/2)\mathrm{Pr}^{-2/3}$ is accurate to a few per cent for air, uncertain by tens of per cent for water, and invalid for liquid metals.